Contribution to thermo fluid modelling of micro heat exchangers using the dimensional analysis

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Abstract: Due to its variety of advantages offered, the progress of the studies on micro devices used in chemical analysis is observed. The micro heat exchangers are well recognized for their higher performance. The applications of them are ranging from increasing of heat transfer applications to chemical reactions or evaporation of liquids applications. The current paper is addressing an engineering approach for modelling the heat and mass transfer processes in micro heat exchangers. The approach is based on the dimensional analysis and principles of theory of similitude that allow the modelling of microscale systems using a physical system at miniscale. There are identified constant relationships between dimensions permitting the analysis of the fluid flow through micro channels, taking into account the differences between fluid flow through micro and mini channels. The velocity scale ratio is in inverse ratio to length scale and similar, it is modified the accelerations scale. The pressure drop is higher with smaller channel dimensions. In this study the interfacial effects are neglected.

Keywords: fluid flow modelling, micro heat exchanger, dimensional analysis, similitude theory.

1. Introduction

The need for new types of heat exchangers that could be applied in various areas such as the cooling of electronic package, bio engineering, advanced energy micro-systems, chemical applications, etc. provided the opportunity for research on flowing phenomena in micro-domain where the channel diameter become extremely small. In this respect, the approach for modelling the heat and mass transfer processes in micro heat exchangers presented in this paper was developed.

Fig. 1. Cross Flow Micro Heat Exchanger realized at Forschungszentrum Karlsruhe

The similitude theory is linking the theoretical aspect of a process with experimental results related to this, creating the physical similitude between model and prototype. The general conditions in studying the similar processes are elaborated. The theory permits the using in practice of theoretical relationships as few dimensionless formulas or to simplify the experimental study of the process by reducing the number of studied describing variables.

A very important component of similitude theory is the dimensional analysis that is a tool for studying the relationships that describe the physical processes. For example, the dimension of a physical quantity is the combination of the basic physical dimensions (usually mass, length, time, and temperature) which describe it; for example, acceleration has the dimension length/square time, and may be measured in meters per square second, miles per square hour, or other units [1]. Dimensional analysis is based on the fact that empirical or rational relations must be dimensionally homogeneous, that means that all terms of a relation have to have the same unit measures.

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2. Theory

The governing differential equations in a process of heat transfer in forced convective single-phase flow, where u is the mean velocity of the fluid in a tube with diameter d, are the followings:

a. *momentum equation:*

$$\rho\frac{\partial u}{\partial t}=-\frac{dp}{dx}+\rho g_x+\frac{\partial}{\partial y}\left(\eta\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\eta\frac{\partial u}{\partial z}\right) \quad (1)$$

b. *enthalpy equation:*

$$\rho c_p\frac{\partial T}{\partial t}=\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)+\rho c_p\Phi \quad (2)$$

The boundary condition is the no-slip condition:

$u=0$ on the boundary. (3)

The variables that *interfere in the physical phenomena* are the followings: mass [kg], pipe length [m], temperature [K], time [s], inner diameter [m], boundary layer depth [m], velocity [m/s], pressure drop [Pa], density [kg/m^3], specific heat of the fluid at constant pressure [J/kgK], dynamic viscosity [Pa·s], cinematic viscosity [m^2/s], thermal diffusivity [m^2/s], thermal conductivity [W/mK], convective coefficient [W/(m^2·K)].

The process of heat transfer in forced convective single-phase flow is directly influenced by 7 variables:

- Inner diameter, d, expressed in dimensional system as [L];
- Velocity, u - in dimensional system: [LT^{-1}];
- Density, ρ - in dimensional system: [ML^{-3}];
- Specific heat of the fluid at constant pressure, c_p - in dimensional system: [$L^2T^{-2}\theta^{-1}$];
- Dynamic viscosity, η - in dimensional system: [$ML^{-1}T^{-1}$];
- Thermal conductivity, λ - in dimensional system: [$MLT^{-3}\theta^{-1}$];
- Convective coefficient, α - in dimensional system: [$MT^{-3}\theta^{-1}$].

3. Results and Discussions

The first step in construction of a scale model is to make an analysis in order to identify what conditions is tested because, despite the fact that the geometry can be uncomplicated scaled, other parameters, such as pressure, temperature or the velocity of fluid might need to be changed. If the testing conditions are created and the test results are applicable to the real design then, similitude is realized. Results from the model can be transferred then without restriction to the original.

As base quantities for the dimensional system is selected the length *L*, mass *M*, time *T* and temperature *θ*. For both scale model and prototype, the values of the dimensionless parameters have to be the same because they have to ensure the similitude.

$$\sigma_L=\frac{l_m}{l} \quad (4)$$

where, σ_L – length scale, l_m – length of model, l – length of prototype.

For example, if the model and prototype are using the same fluid, then, density *[ρ]* expressed in dimensional system as ML^{-3}, dynamic viscosity $[\eta]=ML^{-1}T^{1}$ and thermal conductivity $[\lambda]=MLT^{-3}\theta^{-1}$ are constants, the following relationship could be written:

$$\sigma_\rho=\sigma_M\cdot\sigma_L^{-3}=1\Rightarrow\sigma_M=\sigma_L^3 \quad (5)$$

where, σ_ρ – densities scale, σ_M – masses scale.

$$\sigma_\eta=\sigma_M\cdot\sigma_L^{-1}\cdot\sigma_T^{-1}=1\Rightarrow\cdot$$

$$\sigma_L^3\cdot\sigma_L^{-1}\cdot\sigma_T^{-1}=1\Rightarrow\sigma_T=\sigma_L^2 \quad (6)$$

where, σ_η – dynamic viscosities scale, σ_T – times scale.

$$\sigma_\lambda=\sigma_M\cdot\sigma_L\cdot\sigma_T^{-3}\cdot\sigma_\theta^{-1}=1\Rightarrow$$

$$\sigma_L^3\cdot\sigma_L\cdot\cdot\sigma_L^{-6}\sigma_\theta^{-1}=1\Rightarrow\sigma_\theta=\frac{1}{\sigma_L^2}\cdot \quad (7)$$

where, σ_λ – thermal conductivities scale, σ_θ– temperatures scale.

It is not always possible to realize the strict similitude during a model test, and in several cases some aspects of similitude may be neglected, focusing on only the most important parameters. In this case, the length scale is kept independent. It should be established the determinative regime of process.

For the hydrodynamic processes the viscosity is determinative and the modelling is made based on the similitude condition between model and prototype that involve the equality**:**

$$\mathrm{Re} = \mathrm{Re}_m \tag{8}$$

$$\frac{\rho \bar{u} d}{\eta} = \frac{\rho_m \bar{u}_m d_m}{\eta_m} \tag{9}$$

where *d, d_m* – inner diameter of the pipe for prototype and model, respective in [m], $\bar{u}$, $\bar{u}_m$ – average velocity of the fluid flow for prototype and model, respective in [m/s], ρ , ρ_m - fluid density for prototype and model, respective in [kg/m^3]; η, η_m – dynamic viscosity for prototype and model, respective in [Pa s].

There are noted the scaling ratio with:

$$\sigma_L = \frac{d_m}{d} = \frac{l_m}{l} \tag{10}$$

$$\sigma_\rho = \frac{\rho_m}{\rho} \tag{11}$$

$$\sigma_\eta = \frac{\eta_m}{\eta} \tag{12}$$

$$\sigma_u = \frac{\bar{u}_m}{\bar{u}} \tag{13}$$

There are obtained the following modeling relationships:

$$\sigma_u = \frac{\bar{u}_m}{\bar{u}} = \frac{d}{d_m} \cdot \frac{\eta_m}{\eta} \cdot \frac{\rho}{\rho_m} = \frac{\sigma_\eta}{\sigma_L \sigma_\rho} \tag{14}$$

$$\sigma_{Vf} = \frac{\bar{u}_m A_m}{\bar{u} A} = \frac{\sigma_\eta}{\sigma_L \sigma_\rho} \frac{4\pi d^2{}_m}{4\pi d^2} = \frac{\sigma_\eta \sigma_L}{\sigma_\rho} = \sigma_\nu \sigma_L \tag{15}$$

where $\upsilon=\eta/\rho$ is kinematics viscosity, A and Am is the surface in [m^2], and V_f denotes volumetric flow (flux rate) in [m^3/s].

The quantity σ_u will allow to compute the prototype velocity $\bar{u}$ knowing the model velocity $\bar{u}_m$. Based on this value the mass flux and pressure drop could be calculated.

$$\sigma_{\Delta p} = \frac{\Delta p_m}{\Delta p} = \frac{d}{d_m} \frac{l_m}{l} \frac{\rho_m}{\rho} \left(\frac{\sigma_\eta}{\sigma_L \sigma_\rho} \right)^2 = \frac{\sigma_\eta{}^2}{\sigma_L{}^2 \sigma_\rho} \tag{16}$$

If the model and prototype are using the same fluid, then $\sigma_\rho = 1$ and $\sigma_\eta = 1$. The modelling equations become:

$$\sigma_u = \frac{\bar{u}_m}{\bar{u}} = \frac{1}{\sigma_L} \tag{17}$$

$$\sigma_{Vf} = \frac{\dot{V}_{fm}}{\dot{V}_f} = \sigma_L \tag{18}$$

$$\sigma_{\Delta p} = \frac{\Delta p_m}{\Delta p} = \frac{1}{\sigma_L{}^2} \tag{19}$$

It is marking out that the velocity scale ratio σ_u, is in inverse proportion to the length scale ratio σ_L. The effect is pronounced by maintaining the same volumetric flux rate while scaling down. Particularly, the pressure drop scale becomes huge in microchannels cross section, for the same fluid this enhanced as $\sigma_L{}^2$.

The electrokinetic effects that should occur at the interface between liquids and solids are ignored.

4. Conclusions

Based on the results it is demonstrated that the method of dimensional analysis and principles of similitude theory allow the modeling of system at micro scale using a similar physical system at mini scale, making possible the analysis of flow characteristics through micro channels, but take into consideration the differences between the flows in microscale and miniscale devices.

It is very well known that the optimisation of systems at macro scale integrates microsystems at microscales with optimal working.

In conclusion, the termofluidic processes efficiency at the level of microheatexchangers microchannels is fundamental for enhanced working of systems at macro level.

5. References

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